Limited-Time Offer and Consumer Search

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Abstract

This paper studies a commonly seen but theoretically under-explored sales tactic: limited-time offer. A limited-time offer is any form of discount that a consumer can use on a purchase within a certain period of time. When consumers need time to investigate each product, a firm can endogenously direct the consumer search order by advertising limited-time offers, inducing potential consumers to sample its product early. Whether and how the firm uses limited-time offers depends on the reservation value of its product to the target market. When consumers have only outside options, the firm will use limited-time offers to gain prominence if and only if its reservation value is higher than the outside options. When there are many strategic firms competing for the same target market, the firm with higher reservation value will offer discounts in a shorter time window relative to its competitors, and in equilibrium, it will be sampled earlier by consumers. Contrary to the existing literature, we demonstrate that limited-time offers can increase total welfare through inducing the socially optimal search order.

Keywords: consumer search; limited-time offer; price advertisements; sales tactic

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1 Introduction

A limited-time offer or flash sale is any form of discount that consumers can use when they make a purchase within a certain period of time. Firms can use billboards, television and newspapers to advertise limited-time offers to the general public, or advertise limited-time promo codes to targeted consumer groups via targeted media vehicles such as Facebook or Instagram, as well as send personalized limited-time coupons via email. We can observe in real life that limited-time offers come in a variety of forms, varying not only in magnitude but also in duration, ranging from a few hours to a few days. For example, Figure I and Figure II show two firms' limited-time offers for Black Friday in the same year. Bed Bath & Beyond's offer started at 6 a.m. and lasted until noon of the same day, while IKEA's offer lasted for three days.

Figure I: BB&B's Limited-Time Offer

Figure II: IKEA's Limited-time Offer

Limited-time offers are ubiquitous. More often than not, a firm advertises a discounted price, but also advertises that the discounted price won't last long. Yet, why and how firms develop strategies for limited-time offers is under-explored in the literature. Several behavioral studies have examined consumer psychological reactions to limited-time offers. For example, Inman et al. (1997) present evidence suggesting that "restrictions (i.e., purchase limit, purchase precondition, or time limit) serve to accentuate deal value and act as 'promoters' of promotions."[1] However, the psychological findings do not tell firms what kind of limited-time offer structure they should adopt, such as how much the discount should be and whether it should last longer or shorter relative to other firms.

In this paper, we highlight that consumer search to learn attributes (i.e., is this product right for me) plays an important role in understanding limited-time offers. As shown in Figure III, Forever 21's Black Friday "Early Bird" promotion ended for online shoppers at 9 a.m. and for in-store shoppers at 2 p.m. One plausible explanation for Forever 21 offering different sales windows for online and in-store shoppers is that it takes into account the different amount of time online and offline consumers spend searching. Several empirical studies have also confirmed consumers' search behavior when they receive limited-time offers. For example, Hu et al. (2019) find that after receiving the daily newsletter which covers only the price of the featured deal, Groupon subscribers visit the webpage of the deal and invest time to learn about the deal's features.

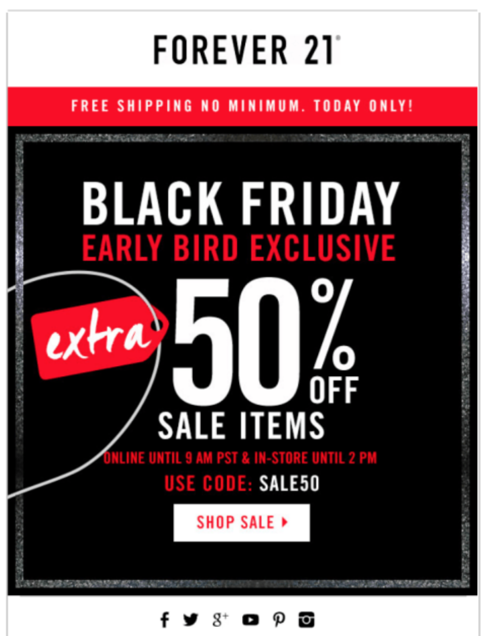

Figure III: Forever 21's Limited-time Offer

[1]Most of the studies consider regret theory (Simonson, 1992) or unavailability theory (Inman et al., 1997) as a behavioral theoretical framework.

As such, this paper delves into firms' use of limited-time offers from a search-related perspective. We suggest that this tactic aims to most effectively gain search prominence, which implies a higher likelihood of consumer purchase.[2] We develop a model of price-directed search with dynamic pricing, considering both the case of a single strategic firm and the case of competing firms. In the first case, we model that a monopolist firm sells a product to target consumers who have outside options. Neither consumers nor the firm knows the actual match values of the product and the outside option ex-ante. The firm can send advertisements containing price and/or promotion information to consumers. Upon receiving the advertisements, consumers choose whether to search the advertised product or the outside option first.

Tactically, to induce search prominence, the monopolist firm can advertise a low uniform price or a limited-time discount, and the latter will capture extra surplus for the firm compared to the former. However, we show that the firm is not always keen on gaining search prominence. It is only most profitable to offer a limited-time discount to gain search prominence if its product's reservation value is higher than the outside option, which is also the condition for the firm being sampled first to be socially optimal.

We further explore limited-time offer strategies in competitive markets. The question is, is the best promotional strategy for all firms to use limited-time offers? If so, how should they design their limited-time offers? How do limited-time offers affect consumers' search behavior in equilibrium?

We begin with a two-period duopoly model in which two firms compete for the same target market. This setting is similar to the monopoly case, except that the outside option becomes a strategic firm. In equilibrium, both firms use limited-time offers, and the search order is socially optimal, i.e., consumers visit first the firm with a superior product (product with higher reservation value). In the case where firms cannot advertise limited-time of-

[2] Following the definition in Armstrong (2017), a product is "prominent" when it is inspected early in consumers' chosen order of search.

fers, a pure-strategy equilibrium does not exist, while a mixed strategy equilibrium exists in which consumers will search in a socially suboptimal order with positive probability. Thus, similar to the finding of the monopolist case, limited-time offers lead to a strict increase in total welfare relative to uniform pricing.

To draw more general conclusions, we extend the market to $N \geq 3$ firms competing over N periods. As in the duopoly market, all the N firms use limited-time offers in equilibrium to invite consumers to sample their products. In terms of the general structure of the offers, all the firms use dynamic discounts. That is, discounts are given at different depths for different time periods, with the depth decreasing over time (i.e., the price increasing over time). Moreover, the firm with a superior product holds sales for a shorter period of time. In equilibrium, consumers' search order coincides with the socially optimal search order. That is, consumers visit the firm with the best product first, then the second best firm, until they visit the worst one. We also extend our setting to a more realistic scenario where consumers' outside options are consumers' private information and not known to firms. The findings of the main models remain robust.

Importantly, our analysis emphasizes the efficient search order as an endogenous outcome of limited-time offers. This is different from the story prevalent in the previous literature, which tells that because consumers cannot observe prices before they search, limited-time discounts are used by firms to hold up consumers who have visited them and are hence anti-competitive. For this reason, Armstrong and Zhou (2015) point out "public policy might, therefore, attempt to limit the use of such tactics." Interestingly, when we imagine a different market context, in a world where prices are transparent (after all, prices can be easily advertised), we come to opposite conclusions and, therefore, opposite policy recommendations. Our findings complement the policy implications regarding limited-time offers, which is another significant contribution of this paper beyond the analysis of the optimal limited-time offer strategy.

The paper unfolds as follows. Section 2 summarizes the related literature. Section 3 describes and solves the monopoly model. Section 4 first introduces the duopoly model, and then extends the market to a more general setting with N firms. Section 5 further considers the case where consumers' outside options are consumers' private information. Finally, Section 6 concludes the paper. Proofs are in the Appendix.

2 Literature Review

Our work builds on consumer search literature. Most of the early search literature assumes that consumers inspect randomly from one option to another to acquire both product and price information (Wolinsky, 1986; Anderson and Renault, 1999). Due to the rapid development of the Internet, price information is now easily accessible to consumers, and firms can proactively advertise prices to their target customers as well. The observability of prices before search has inspired a new strand of literature that studies situations where consumer search is influenced by firms' prices. While most of the models studying price-directed search are static (Armstrong and Zhou, 2011; Haan et al., 2018; Choi et al., 2018; Ding and Zhang, 2018), we explore firm pricing and consumer search in a dynamic framework.

Table I summarizes the differences between our paper and the above-cited papers. In addition, the paper that is closest in spirit to ours is Armstrong and Zhou (2015). They model a random-search process during which firms use exploding offers or buy-now discounts to dissuade consumers who have visited their stores from going elsewhere. In contrast, in our setting, firms advertise limited-time offers, and thus, consumers can observe product prices and strategically decide the search order among firms. Interestingly, these two stories, which do not seem so different at first glance, have opposite results. We show that limited-time offers can increase total welfare through inducing a socially optimal search order *endogenously*, while they find that exploding offers and buy-now discounts may lead to welfare losses for an *exogenous* search order.

Another related section of the search literature focuses on situations where the search or-

Table I: Papers on Consumer Search

Search Order \ Pricing	Static Pricing	Dynamic Pricing
Random Search	Wolinsky (1986) Anderson and Renault (1999)	Armstrong and Zhou (2015)
Price-directed Search	Armstrong and Zhou (2011) Choi et al. (2018) Haan et al. (2018)	This paper

der is not random but determined by firms' non-pricing features and strategies. Arbatskaya (2007) shows that when consumer search order is exogenously determined, there is price dispersion among firms selling homogeneous goods, and prices and profits decline in the order of search. Armstrong et al. (2009) study a search market with prominence in which all consumers sample the prominent firm first and randomly search among remaining firms if the prominent product is not satisfactory. Armstrong and Zhou (2011) discuss several ways by which firms can become prominent. Chen and He (2011), Haan and Moraga-González (2011) and Athey and Ellison (2011) assume that the order in which firms are visited is influenced by advertising expenditures or online bidding in position auctions. Janssen and Ke (2020) model that service provision leads to prominence in consumers' search process. Wang et al. (2021) study intrabrand competition and find that prominence can increase or reduce a retailer's profit. In this paper, we find a similar equilibrium result as in Armstrong et al. (2009) that the firm being sampled first may charge a lower price than others.[3]

More broadly, this paper is related to the extensive literature that studies discounts and coupons (Narasimhan, 1984; Shaffer and Zhang, 1995, 2002; Banks and Moorthy, 1999). In this literature, coupons are mainly considered as a tool of price discrimination. In particular, Banks and Moorthy (1999) introduce search cost into their price discrimination model of promotions. Unlike the search process described in this paper, their framework is close to that of Varian (1980) and Narasimhan (1988), but with a new assumption that promotional prices are only available when offered and only for those who search for them.

[3]See also Armstrong (2017) for a comprehensive introduction of the literature on ordered search.

3 Monopoly Model

We first consider a monopoly model in which a representative consumer of the target market chooses between the monopolist's product and a non-strategic outside option. For example, imagine a consumer who, after reading some travel blogs online, receives a promotional email from a travel agency. The email ad includes a promo code for a trip to New York that the consumer can use to receive a discount if she books within one day. A trip to New York with the agency is one option; alternatively, the consumer can plan a trip to another city on her own. Both options require time and effort to research, and the consumer needs to consider which option to investigate first. In this section, we will look for the best promotional strategy for the monopolist.

3.1 Model Setting

The monopolist offers a product (*option* A) at a marginal cost normalized to zero and competes with the non-strategic outside option (*option* B) for the representative consumer's unit demand. The product's match value to the consumer is ex-ante uncertain and follows a two-point distribution: $v_A > 0$ (good match) with probability $\theta_A \in (0, 1)$ and 0 (bad match) with probability $1 - \theta_A$. The outside option yields an uncertain net surplus to the consumer, which also follows a two-point distribution: $v_B > 0$ with probability $\theta_B \in (0, 1)$ and 0 with probability $1 - \theta_B$. The match value realizations of A and B are independent.[4]

The search and purchase process takes at most two periods, $t = 1, 2$. A period can be interpreted as the approximate time it takes for consumers to complete a visit to a firm from the time they receive an advertisement from the firm. All agents are risk-neutral and do not discount future payoffs.[5] The timing of the game is as follows. The monopolist sends

[4]Similar valuation distributions have appeared in some recent search models, such as Anderson and Renault (2017), Ding and Zhang (2018), and Preuss (2021).

[5]We consider this assumption to be realistic as most real-world limited-time offers last a brief time. The results will not change qualitatively for other discount factors in between 0 and 1.

advertisements to its target market informing about its prices and discounts (if any) for the next two periods. From $t = 1$, the consumer can at most sample one option per period by incurring a search cost s. If the consumer samples one option at $t = 1$, she finds out its match value and decides whether to purchase there or continue to search. If the consumer continues to search at $t = 2$, she can buy the newly-explored option or buy the previously sampled option at the price of $t = 2$ without extra search cost. If the consumer purchases neither the product nor the outside option, her payoff is zero.

The tie-breaking rules are specified as follows. First, when the consumer is indifferent between searching (purchasing) or not, she searches (purchases). Second, when the consumer is indifferent between two search orders, she chooses the search order that maximizes the expected total welfare. Third, when all the search orders generate the same expected total welfare, the consumer randomly chooses one search order.[6] Last, to avoid trivial cases, we assume $\min\{\theta_A v_A, \theta_B v_B\} > s$, meaning that the other option is worth investigating if the first-sampled option turns out to be a bad match.

3.2 The First Best

To begin with, we look for the first-best search order which maximizes total welfare regardless of price (which is simply a transfer of surplus from consumers to firms).[7] The optimal search path could be characterized by two features: first, it is optimal for the consumer to continue to search if the previously sampled option is a bad match, due to the assumption $\min\{\theta_A v_A, \theta_B v_B\} > s$; second, if the consumer samples option i first and i turns out to be a good match, the consumer will not proceed to option j, because otherwise, it is socially optimal to sample option j first.

The search order can be from A to B or from B to A. Next, we compare the total welfare

[6] These are standard assumptions to avoid epsilon-equilibrium. The result will not change qualitatively under other tie-breaking rules.

[7] Note that this is a simple case of Weitzman (1979).

under each search order. The total welfare under the search order from A to B is:

$$W_{A\to B} = \theta_A v_A - s + (1-\theta_A)(\theta_B v_B - s)$$

and the total welfare under the search order from B to A is:

$$W_{B\to A} = \theta_B v_B - s + (1-\theta_B)(\theta_A v_A - s)$$

It is immediate that $W_{A\to B} > W_{B\to A}$ if and only if $v_A - \frac{s}{\theta_A} > v_B - \frac{s}{\theta_B}$. Note that, the expected consumer surplus from searching and purchasing product i is $\theta_i(v_i - p) - s$, so $v_i - \frac{s}{\theta_i}$ is the **reservation value** for product i, which is also the maximum price the monopolist can charge, and therefore the **monopoly price**. We denote $v_i - \frac{s}{\theta_i}$ by $\mathbf{RV_i}$ hereafter. It is socially optimal for the consumer to start her search with the option that has a higher reservation value.

3.3 The Monopolist's Optimal Pricing Strategy

In this section, we solve for the monopolist's optimal pricing and promotion strategy and the market equilibrium. Again, there are two possible search orders: the consumer samples the monopolist's product first, or the outside option first. Regardless of which option is sampled first, consumers do not recall in equilibrium. This finding is summarized in the following lemma.

Lemma 1 (Consumer Search Rule) *Suppose that the consumer samples option i first $(i \in \{A, B\})$. The consumer will not continue to search if option i turns out to be a good match, implying that the consumer never recalls in equilibrium.*

The proof is provided in Appendix A. Intuitively, if a consumer chooses to continue to search after finding that the first product is a good match, then she should have started with another product first.

Next, we compare the maximum profits that the monopolist can obtain under two different search orders and the corresponding pricing strategies. Without loss of generality, we can restrict the range of prices in equilibrium to $p_A \in [0, v_A - \frac{s}{\theta_A}]$. This is because the **monopoly price**, $v_A - \frac{s}{\theta_A}$, is the highest price that makes consumers indifferent between searching and not searching when the product is the only option. Any price above the monopoly price will deter consumers from searching. We classify the potential optimal pricing strategies into three types: monopoly price, uniform low price and limited-time offer.

Monopoly Price The first pricing strategy we consider is to charge the monopoly price in both periods. Such pricing yields the highest profit given that the monopolist's product is sampled at $t = 2$. The expected profit is $\hat{\pi} = (1 - \theta_B)(\theta_A v_A - s)$, where $(1 - \theta_B)$ denotes the probability that the outside option is a bad match.[8]

Uniform Low Price Another strategy is to offer a uniform low price. For example, the monopolist could send advertisements to consumers informing them of a new price that is discounted from the original (monopoly) price, and that the new price is not limited-time. With a higher expected surplus, consumers may choose to visit the monopolist first. To induce the search order $A \to B$, the uniform price, denoted by p_u, should satisfy the following incentive compatibility constraint on the consumer:

$$\theta_A(v_A - p_u) - s + (1 - \theta_A)(\theta_B v_B - s) \geq \theta_B v_B - s + (1 - \theta_B)[\theta_A(v_A - p_u) - s]$$

The LHS is the expected surplus from searching in the order of $A \to B$. The consumer samples option A first at a search cost s, and with a probability θ_A, finds a good match which yields a payoff $(v_A - p_u)$. If option A turns out to be a bad match, the consumer continues to sample option B at another cost s, and with a probability θ_B, the consumer finds option B a

[8]Note that this pricing strategy is equivalent in expected profit to any other price plan consisting of a first-period price that makes consumers sample outside option first and the monopoly price in the second period.

good match which yields a net surplus v_B. The RHS is the expected surplus from searching in the order of $B \rightarrow A$.

The optimal uniform price to induce search order $A \rightarrow B$ is $\tilde{p}_u = v_A - \frac{s}{\theta_A} - v_B + \frac{s}{\theta_B}$, a $(\frac{RV_B}{RV_A}) * 100\%$ discount off the monopoly price. The expected profit using this strategy is $\tilde{\pi}_u = \theta_A \tilde{p}_u$.

Limited-Time Offer Alternatively, the monopolist can set a relatively low price p_1 that lasts for only one period, and a higher price p_2 in the second period. The purpose is like the previous strategy, to induce consumers to sample its product first and then buy the product if it turns out to be a good match. The monopolist aims to maximize $\pi = \theta_A p_1$ under the consumer's incentive compatibility constraint:

$$\theta_A(v_A - p_1) - s + (1-\theta_A)(\theta_B v_B - s) \geq \theta_B v_B - s + \max\{0, (1-\theta_B)[\theta_A(v_A - p_2) - s]\}$$

The monopolist's optimal price at $t = 1$ is then $\bar{p}_1 = v_A - \frac{s}{\theta_A} - (\theta_B v_B - s)$. The optimal second-period price should be a price that exploits all the consumer surplus at $t = 2$, providing room for setting a high price in the first period, i.e., $\bar{p}_2 = v_A - \frac{s}{\theta_A}$. Such price plan, consisting of a low buy-now price and a high buy-later price, is a **limited-time offer**. With a limited-time offer, the consumer visits the monopolist first and buys there if the product's match value is high. The monopolist's profit is $\bar{\pi} = \theta_A \bar{p}_1$.

The Optimal Pricing Strategy First, it is straightforward to obtain that $\bar{p}_1 > \tilde{p}_u$ and $\bar{\pi} > \tilde{\pi}_u$. It is not surprising that dynamic pricing (limited-time offer strategy) *strictly* dominates uniform pricing (uniform low price) in inducing the same search order $A \rightarrow B$. In the case of limited-time offer, setting a high buy-later price is equivalent to increasing the opportunity cost of sampling the outside option B first, which relaxes the constraint on the monopolist to set a low price in the first period. Therefore, $\bar{p}_1 > \tilde{p}_u$.

Then, by comparing the profits of limited-time offer and monopoly pricing, $\bar{\pi}$ and $\hat{\pi}$

respectively, we find that $\bar{\pi} > \hat{\pi}$ if and only if $RV_A > RV_B$. The following proposition summarizes the result:

Proposition 1 *When the product has a higher reservation value than the outside option, using a limited-time offer to gain search prominence is the most profitable strategy for the monopolist. The limited-time offer consists of a* $(\theta_B \frac{RV_B}{RV_A}) * 100\%$ *discount off the monopoly price for the first period and the monopoly price for the second period. In the opposite case, it is optimal for the monopolist to set the monopoly price* $p = RV_A$ *and its product will be sampled in the second period.*

From the proposition, we can see that the monopolist is not always keen on gaining search prominence. Offering a limited-time discount is the optimal pricing strategy if and only if the product's reservation value is higher than the outside option, which is also the condition for the socially optimal search order to start with the firm. The intuition is that the limited-time offer strategy allows the monopolist to extract all the surplus net of what the outside option provides, and thus maximizing the total welfare is equivalent to maximizing the firm's profit.

We can also compare the profits of monopoly pricing and uniform low price strategy. When $RV_B > \theta_B \cdot RV_A$, we have $\hat{\pi} > \tilde{\pi}_u$, i.e., setting monopoly price is more profitable than uniform low price. It is because when the outside option is very competitive, the monopolist needs to provide a high discount ($\frac{RV_B}{RV_A} * 100\%$) to lure consumers to sample its product first. Too high a discount makes the prominent search position not worthwhile and the monopolist prefers to choose monopoly pricing and be sampled in the second period. The above analysis implies that when $RV_A > RV_B > \theta_B \cdot RV_A$, if sending limited-time offers to the target market is not feasible, the firm will use monopoly pricing and the resulting search order is socially suboptimal. We summarize these findings below.

Corollary 1 *The monopoly market equilibrium achieves the first-best market allocation in the presence of limited-time offers. If limited-time offers were prohibited or infeasible, the monopolist may induce consumers to search in an inefficient order.*

4 Competitive Limited-Time Offer Strategies

In real life, it is also common to see firms simultaneously advertising limited-time offers to compete for the same target market, especially during traditional shopping holidays like Black Friday. An example is the competition between Bed Bath and Beyond and IKEA, as shown in Figure I and II. In this section, we aim to explore firms' optimal limited-time offer strategies in the presence of market competition. To provide a more detailed exposition, we first solve for the equilibrium of a duopoly market and then extend the setting to a competitive market with N firms ($N \geq 3$) to draw more general conclusions.

4.1 Duopoly Model

Two firms compete for the same consumer group with products A and B, respectively. The match value of product A to the representative consumer is a random variable that takes the value $v_A > 0$ (good match) with probability $\theta_A \in (0,1)$ and the value 0 (bad match) with probability $1 - \theta_A$. The match value of product B also follows a two-point distribution and takes the value v_B with probability $\theta_B \in (0,1)$ and the value 0 with probability $1 - \theta_B$. Both firms inform consumers of their pricing and discount plans through advertisements. Without loss of generality, we assume $RV_A \geq RV_B > 0$. As there are only two products and consumers spend at most two periods sampling both, we maintain the assumption that the market operates for two periods in the duopoly model. In the next section with N firms ($N \geq 3$), the assumption is relaxed and the market operates for N periods.

Let $\{(\hat{p}_1^A, \hat{p}_2^A), (\hat{p}_1^B, \hat{p}_2^B)\}$ hereafter denote the equilibrium prices of product A and B in periods 1 and 2. Before deriving the equilibrium, we list all the five possible consumer search paths for any given prices advertised by the two firms. The possible paths are:

Path I ($A \rightarrow B$): at $t = 1$, the consumer samples product A. If product A is a good match (of match value v_A), the consumer purchases it at price p_1^A; if product A is a bad match (of match value 0), the consumer continues to search at $t = 2$ to find out the match value of product B. The consumer only purchases product B at price p_2^B if it is a good match (of match value v_B).[9]

*P*ath II ($B \to A$): at $t = 1$, the consumer samples product B. If product B is a good match, the consumer purchases it at price p_1^B; if product B is a bad match, the consumer continues to search at $t = 2$ to find out the match value of product A. The consumer only purchases product A at price p_2^A if it is a good match.

*P*ath III ($\emptyset \to A$ or $\emptyset \to B$): at $t = 1$, the consumer does not search. The consumer samples either product A or product B at $t = 2$.

*P*ath IV ($A \to \emptyset$ or $B \to \emptyset$): at $t = 1$, the consumer samples either product A or product B. Independent of the search result, the consumer does not search at $t = 2$.

*P*ath V ($\emptyset \to \emptyset$): the consumer never searches. It is easy to see that this path does not constitute any equilibrium nor deviation, as this path yields zero profit for both firms.

4.1.1 Equilibrium Search Path and the Profit-dominant Equilibrium

Path III or IV can occur if prices in the first or second period are too high. However, neither of them sustain an equilibrium because the non-sampled firm at the non-sampled period can deviate to a lower price to induce search. Path I or II could possibly be the equilibrium search path, and any equilibrium with Path I or II must satisfy the properties stated in the following lemma:

Lemma 2 *Suppose there is a pure-strategy Nash equilibrium with prices $\{(\hat{p}_1^i, \hat{p}_2^i), (\hat{p}_1^j, \hat{p}_2^j)\}$, $i, j \in \{A, B\}, i \neq j$, in which the consumer searches following the path $i \to j$. Then it must be that $\hat{p}_2^j = v_j - \frac{s}{\theta_j} \equiv RV_j$, and $\hat{p}_2^i = v_i - \frac{s}{\theta_i} \equiv RV_i$.*

Proof. Note that RV_j is the highest price that makes consumers willing to sample firm j.

[9] For any given price combinations, it can never be optimal that the consumer samples product B after A regardless of product A's match value. It is strictly dominated by a search strategy of sampling B first and only sampling A if B is a bad match. More details can be found in the proof of Lemma 1 in Appendix A.

For any $\hat{p}_2^j < RV_j$, a profitable deviation for firm j is to increase $\hat{p}_2^j$ to RV_j and increase $\hat{p}_1^j$ if necessary to maintain the search order $i \to j$. Therefore, it must be that $\hat{p}_2^j = v_j - \frac{s}{\theta_j} \equiv RV_j$.

Suppose $\hat{p}_2^i < RV_i$, to make the consumer follow search path $i \to j$, it must hold that:

$$\theta_i(v_i - \hat{p}_1^i) + (1-\theta_i)[\theta_j(v_j - \hat{p}_2^j) - s] \geq \theta_j(v_j - \hat{p}_1^j) + (1-\theta_j)[\theta_i(v_i - \hat{p}_2^i) - s] \tag{1}$$

where $\theta_j(v_j - \hat{p}_2^j) - s = 0$ because we have proved $\hat{p}_2^j = RV_j$, and $\theta_i(v_i - \hat{p}_2^i) - s > 0$ by assumption. Therefore, a profitable deviation is to raise $\hat{p}_2^i$ to $\tilde{p}_2^i = RV_i$ and simultaneously raise $\hat{p}_1^i$ while keeping the above inequality satisfied. Hence, we can conclude that $\hat{p}_2^i = v_i - \frac{s}{\theta_i} \equiv RV_i$. ■

Next, as we show in the following proposition, Path II cannot constitute an equilibrium when $RV_A > RV_B$:

Proposition 2 (Equilibrium Search Path) *When $RV_A > RV_B$, the only possible search path in all pure-strategy equilibria is $A \to B$ (Path II): the consumer first samples product A and continues to sample product B if product A is a bad match.*

Proof. Suppose there exists an equilibrium with prices $(\hat{p}_1^i, \hat{p}_2^i), (\hat{p}_1^j, \hat{p}_2^j)$ and search path $i \to j$. Lemma 2 implies that in either $A \to B$ or $B \to A$ path, the consumer surplus at $t = 2$ is zero in expectation. Therefore, the consumer follows the $i \to j$ search path if and only if:

$$\theta_i(v_i - \hat{p}_1^i) - s \geq \theta_j(v_j - \hat{p}_1^j) - s \tag{2}$$

When $\hat{p}_1^j \leq RV_j$, the above inequality must be binding in equilibrium, otherwise firm i can slightly increase $\hat{p}_1^i$. Hence, we can rearrange the consumer's IC constraint (2) to:

$$\hat{p}_1^j = v_j - \frac{\theta_i}{\theta_j}(v_i - \hat{p}_1^i) \tag{3}$$

Given the candidate equilibrium price combination, firm j makes a profit of $(1-\theta_i)\theta_j RV_j$ in equilibrium. Since Equation (3) implies that the consumer is ex-ante indifferent between $i \to j$ and $j \to i$ paths in term of expected payoff, a slight decrease in $\hat{p}_1^j$ would lead the consumer to switch to $j \to i$ search path. To avoid such deviation, we need $\theta_j \hat{p}_1^j \leq (1-\theta_i)\theta_j RV_j$. Replace $\hat{p}_1^j$ with Equation (3), we have:

$$\hat{p}_1^i \leq v_i - \frac{s}{\theta_i} - \theta_j(v_j - \frac{s}{\theta_j}) \tag{4}$$

We also need to find conditions that ensure no deviations from firm i. It could be possible that firm i find it more profitable to increase $\hat{p}_1^i$ and set $\hat{p}_2^i = RV_i$ to be sampled second. The maximum profit that firm i can make in the $j \to i$ search path is $(1-\theta_j)\theta_i RV_i$. For the deviation to be non-profitable, it must hold that $\theta_i \hat{p}_1^i \geq (1-\theta_j)\theta_i RV_i$, which can be simplified to:

$$\hat{p}_1^i \geq v_i - \frac{s}{\theta_i} - \theta_j(v_i - \frac{s}{\theta_i}) \tag{5}$$

Given the parameter assumption of $v_A - \frac{s}{\theta_A} > v_B - \frac{s}{\theta_B}$, condition (4) contradicts with condition (5) if $i = B$ and $j = A$. Therefore, there does not exist a pure-strategy Nash equilibrium in which the consumer follows the $B \to A$ search path. ■

Thus far, we have shown that the only possible search path in the equilibrium is $A \to B$: the consumer samples product A first at $t = 1$, and if product A turns out to be a bad match, the consumer samples product B at $t = 2$. Next, we will characterize the pure-strategy **profit-dominant equilibrium**, i.e., the Nash equilibrium in which both firms obtain their highest profits among all pure-strategy equilibria. This is a natural equilibrium selection because consumers are price-takers, and firms in the market are typically long-lived and repeatedly playing against each other (Harsanyi et al., 1988). A profit-dominant equilibrium is denoted by $\{(p_1^{A*}, p_2^{A*}), (p_1^{B*}, p_2^{B*})\}$.

According to Lemma 2, we know that the profit of firm A is $\theta_A \hat{p}_1^A$ and that of firm B is

$(1-\theta_A)\theta_B\hat{p}_2^B$ where $\hat{p}_2^B = v_B - \frac{s}{\theta_B}$ in all pure-strategy equilibria. Hence, a profit-dominant equilibrium must entail the highest possible $\hat{p}_1^A$ which is given by Inequality (4), that is,

$$p_1^{A*} = v_A - \frac{s}{\theta_A} - \theta_B(v_B - \frac{s}{\theta_B})$$

Finally, from Equation (3), we find that the corresponding p_1^{B*} is given by:

$$p_1^{B*} = (1-\theta_A)(v_B - \frac{s}{\theta_B})$$

The following proposition characterizes the profit-dominant equilibrium:

Proposition 3 (Profit-dominant Equilibrium) *When $RV_A > RV_B$, the pure-strategy profit-dominant equilibrium exists and is characterized by: $\{(p_1^{A*}, p_2^{A*}), (p_1^{B*}, p_2^{B*})\}$, where $p_1^{A*} = v_A - \frac{s}{\theta_A} - \theta_B(v_B - \frac{s}{\theta_B})$, $p_2^{A*} = v_A - \frac{s}{\theta_A}$, $p_1^{B*} = (1-\theta_A)(v_B - \frac{s}{\theta_B})$, $p_2^{B*} = v_B - \frac{s}{\theta_B}$. In this equilibrium, the consumer first samples product A at $t = 1$, and only if product A is not a good match, the consumer samples product B at $t = 2$. When $RV_A = RV_B$, the search order can be either $A \to B$ or $B \to A$ in equilibrium, and both firms make the same profit.*

Compare this equilibrium with the case where the competitor is not strategic (monopoly model). Firm A sets the same price as when competing with a non-strategic outside option. When the outside option becomes a strategic firm, the firm also offers a limited-time discount in equilibrium. The effective transaction prices of product A and product B are p_1^{A*} and p_2^{B*} respectively. When $RV_B < RV_A < (1+\theta_B) \cdot RV_B$, product A is sold at a cheaper price despite a higher reservation value ($p_1^{A*} < p_2^{B*}$). The reason is that in this price-directed search market, firm A competes hard in price to earn search prominence. Meanwhile, firm B is able to charge a monopoly price because firm B is essentially a monopolist for the consumer who fails to find a good match in product A. Therefore, even though product A has a higher reservation value, its final price might be lower due to the competition. It is worth noting that although this result seems to be similar to Armstrong et al. (2009), the mechanism is quite different. In Armstrong et al. (2009), the prominent firm sets a lower price as it faces a more elastic demand. In our model, the demand is inelastic, and firm A charges a lower price due to the endogenous search order.

In the profit-dominant equilibrium, both firms obtain the highest profits among all pure-strategy Nash equilibria: $\pi^{A*} = \theta_A(v_A - \frac{s}{\theta_A} - \theta_B v_B + s)$ and $\pi^{B*} = (1-\theta_A)\theta_B(v_B - \frac{s}{\theta_B})$. In terms of the market share and firm's profit, firm A might also do worse than firm B when $\theta_A \cdot RV_A < \theta_B \cdot RV_B$. It may occur when the matching probability of product B is (much) higher than product A: $\theta_B > \theta_A$. Note that, this result is different from the predictions of ordered search literature (Arbatskaya, 2007; Armstrong et al., 2009; Zhou, 2011; Chen and He, 2011) that the prominent firm enjoys a larger market share and profit. In our setting, firms are heterogeneous ex-ante and prominence in the equilibrium is endogenously determined. The firm that obtains prominence could have a lower probability of being a good match, and its prominent position may be accompanied by a lower transaction price.

We are also interested in analyzing the effect of search cost on the market price. It is easy to see that the equilibrium prices for both firms fall when the search cost rises, which is in line with the past literature on price-directed search (Haan et al., 2018; Ding and Zhang, 2018; Choi et al., 2018). The non-prominent firm (firm B) extracts all the surplus from the consumer's second visit conditional on the first product being a bad match, and thus a higher search cost reduces the surplus that firm B could extract. For the prominent firm (firm A), the search cost affects p_1^{A*} directly and indirectly. The direct effect is reflected in the "$-\frac{s}{\theta_A}$" part. A higher search cost reduces the consumer's total surplus from searching, and therefore, firm A needs to reduce the price. The indirect effect is reflected in the "$-\theta_B(v_B - \frac{s}{\theta_B})$" part. The direct effect causes a lower first-period price of product A, and thus the profit in firm B's most profitable deviation is smaller. Therefore, firm B is less likely to undercut firm A, which allows the equilibrium price of A to be higher. Overall, the direct effect dominates the indirect effect for firm A, and the equilibrium price of product A also falls with a higher search cost.

4.1.2 Welfare Analysis

In the monopoly model, we show that the market equilibrium may not follow the optimal search order if limited-time offers are prohibited or restricted. So what are the relevant policy implications when there are two competing firms in the market? To answer this question, we need to know the market equilibrium if only uniform pricing is possible. We prove the following lemma in Appendix A.

Lemma 3 *No pure-strategy Nash equilibrium exists under uniform pricing.*

Nevertheless, there exists a mixed-strategy equilibrium under the uniform pricing scheme, the solution of which is presented after the proof of Lemma 3 in Appendix A. In the mixed-strategy equilibrium, the firms are visited in the socially suboptimal order $B \rightarrow A$ with positive probability. Therefore, as summarized in Proposition 4, allowing for limited-time offers strictly increases the total welfare.

Proposition 4 (Welfare) *When firms compete in limited-time offers, compared to a market under uniform pricing, limited-time-offer equilibrium achieves the socially optimal search order and a higher total welfare.*

This result echoes Corollary 1, but is contrary to the prevailing view in the literature. Armstrong and Zhou (2015) find that exploding offers and buy-now discounts lead to welfare loss. In their model, buy-now discounts are used after consumers visit the firm, so such sales tactics are associated with hold-up and anti-competitive behavior, which may lead to a decrease in buy-later demand and in the quality of the match between product and consumer. In our model, limited-time offers are advertised to consumers before they search; market competition endogenously directs consumers to search in an efficient order. Therefore, rather than being anti-competitive, limited-time offers are pro-competitive.

4.2 Competitive Market with N Firms

In this section, we extend the duopoly model to a general case where N firms ($N \geq 3$) compete for a target market at the same time. We will show that under slightly stronger conditions, our findings of the previous sections persist.

Suppose N firms (firm $1, 2$...and N) are targeting the same group of consumers. Each firm has a product and the match value of the product to the representative consumer is binary: a good match of value v_i with probability θ_i, or a bad match of value 0 with probability $(1-\theta_i), i = 1, 2 \ldots N$. The reservation value for firm i is $RV_i = v_i - \frac{s}{\theta_i}$. W.l.o.g, let $RV_1 \geq RV_2 \geq \cdots \geq RV_N$. The market operates for N periods. Each firm i advertises its price plan $\{p^i_t\}_{t=1,2\ldots N}$ to consumers. After observing the price plans of all firms, the consumer starts a sequential search process. Details on how to solve for the market equilibrium and how to prove its properties can be found in Appendix A. We summarize the results below:

Proposition 5 *With N competing firms in the market, we have:*

1. *All the pure-strategy equilibria are socially efficient, i.e., $\forall i, j \in 1 \ldots N$, if $RV_i > RV_j$, the consumer samples product i before j.*

2. *When firm 1 to N can be ranked by first order stochastic dominance (FOSD), there exists a profit-dominant equilibrium. For any firm $i < N$,*

$$p^i_t = \begin{cases} (1-\theta_{i-1})p^i_i & \text{if } t < i \\ RV_i - \sum_{k=i+1}^{N} \left[\theta_k RV_k \Pi_{m=1}^{k-1}(1-\theta_m)\right] & \text{if } t = i \\ RV_i & \text{if } t > i \end{cases}$$

For firm N,

$$p^N_t = \begin{cases} (1-\theta_{N-1})RV_N & \text{if } t < N \\ RV_N & \text{if } t = N \end{cases}$$

The consumer samples products sequentially from firm 1 *to firm* N *and buys the first product that is found to be a good match.*[10]

Figure IV below illustrates a numerical example of the price equilibrium with six competing firms. Each time a firm is sampled, the consumer bears the search cost $s = 1$. All six firms have the same good-match value: $v_i = 10$, but they differ in the probability of being a good match: $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = (0.8, 0.7, 0.6, 0.5, 0.4, 0.3)$. Thus, in the sense of FOSD, firm 1 dominates firm 2, firm 2 dominates firm 3, and so on.[11]

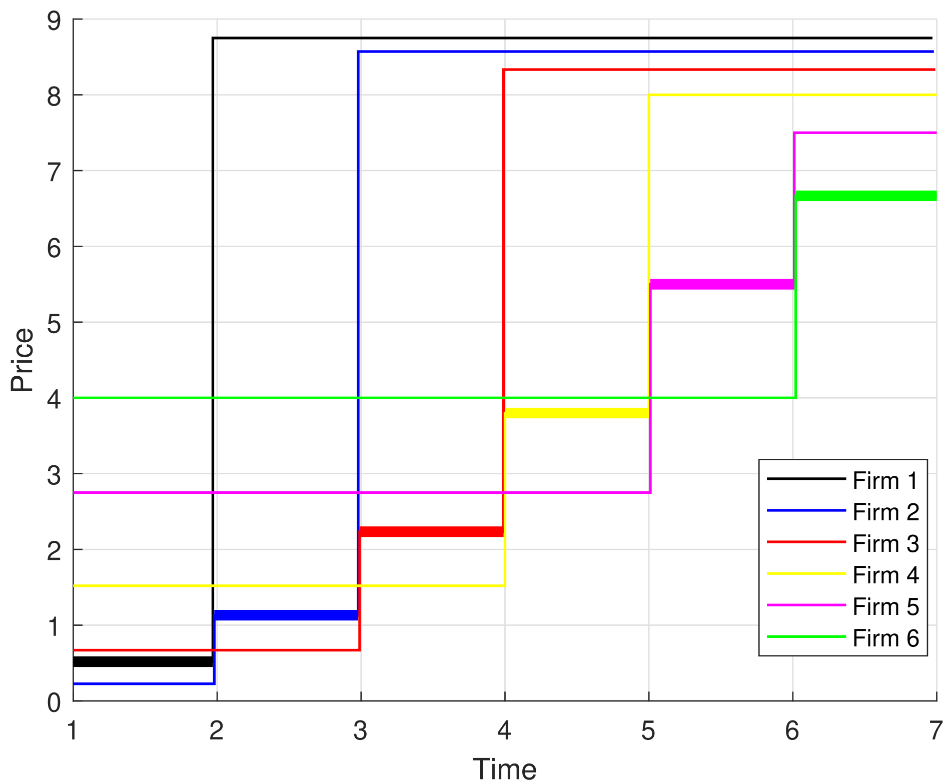

Figure IV: A numerical example of a market with six firms

[10]The stronger assumption that all firms can be ranked by FOSD is made to obtain an analytical solution for the profit-dominant equilibrium. In addition, there exist many equilibria with the same transaction price and search order. The equilibrium described in the proposition is the equilibrium with the smallest price change for all firms, which would naturally be selected if any negligible positive menu cost is assumed.

[11]FOSD implies that the firm with a higher reservation value also owns a higher probability of being a good match. The assumption includes two special cases: first, all the firms have the same probability of being a good match ($\theta_i = \theta, \forall i$), but differ in the good-match value ($v_1 \geq v_2 \geq \cdots \geq v_N$); second, all the firms have the same good-match value ($v_i = v, \forall i$), but differ in the probability of being a good match ($\theta_1 \geq \theta_2 \geq \cdots \geq \theta_N$).

From Figure IV, we can clearly observe two characteristics of the price equilibrium stated in Proposition 5. First, all the firms use dynamic discounts, and the depth of the discount (relative to the monopoly price) decreases over time. For example, firm 2 uses a deeper discount in the first period than in the second period, reverts to the monopoly price from the third period, and maintains the monopoly price until the final period. Second, firms that are perceived by their target consumers to be stronger ex-ante (in the sense of FOSD) hold discounts for a shorter period of time and revert to their monopoly prices earlier. In equilibrium, consumers will sample the products in the order from firm 1 to firm N. The effective transaction price (i.e., the price of a product at the time it is sampled) for each firm is highlighted as a thick line. As shown in the figure, in this numerical example, the effective transaction price increases over time.

The results for the general setup of N firms suggest that our previous findings are robust. The first part of Proposition 5 parallels Proposition 2. In all pure-strategy equilibria, firms that are perceived to be better before search are sampled earlier by consumers, which coincides with the socially optimal search order. The reason is that firms with lower reservation value must give larger discounts to gain search prominence against stronger firms. As a result, stronger firms have a greater incentive to compete for prominence, while the best strategy for weaker firms is to wait until consumers have visited those stronger firms.

Moreover, it is straightforward that Lemma 3 also hold when there are $N \geq 3$ firms. That is, if firms are only allowed to adopt uniform pricing, there exists no pure-strategy equilibrium with search order $1 \to 2 \to \cdots \to N$. When there are more than two firms competing, allowing firms to use limited-time offers in competition *strictly* increases total welfare, echoing Proposition 4.

5 Extension: Private Outside Options

In this section, we propose an extension of the monopoly model to examine a possible case in reality where outside options of consumers are private information and are not observed by the firm. For example, following up on the travel agency example we mentioned earlier, consumers may be heterogeneous in their ability to plan trips, but travel agencies cannot observe consumer characteristics in order to tailor limited-time offers. Only consumers themselves know how likely it is that an outside option will be a good match, and how much utility can be gained from a good match.

We assume that the representative consumer's outside option (v_B, θ_B) (also denoted as the consumer's type) is drawn from a joint distribution with p.d.f $f(v,\theta)$, where $v > 0$ and $\theta \in (0,1)$ for all $(v,\theta) \in \mathrm{supp}(f)$. (v_B, θ_B) is privately observed by the consumer and unknown to the firm. We assume that even for the worst possible draw, the outside option is worth investigating, i.e., $v - \frac{s}{\theta} > 0, \forall (v,\theta) \in \mathrm{supp}(f)$. The following proposition summarizes our findings:

Proposition 6 (Private Outside Option) *Let $\underline{U} = \min \theta v, \forall (v,\theta) \in supp(f)$. Let $\mathcal{B}$ denote the set of $(v,\theta) \in supp(f)$ such that $\theta v = \underline{U}$. We have:*

1. *If $\forall (v,\theta) \in supp(f)$, $v_A - \frac{s}{\theta_A} \leq v - \frac{s}{\theta}$, it is optimal for the monopolist to charge a uniform price at $p = v_A - \frac{s}{\theta_A}$ and always being sampled in the second period.*

2. *If $\forall (v,\theta) \in \mathcal{B}$, $v_A - \frac{s}{\theta_A} > v - \frac{s}{\theta}$, limited-time offer is strictly more profitable than uniform pricing, and the monopolist is sometimes sampled first in equilibrium.*

In the optimal limited-time offer, the monopolist sets a high buy-later price that extracts all reservation value ($p_2 = v_A - \frac{s}{\theta_A}$) and a low buy-early price p_1.

A formal proof is provided in Appendix A. Proposition 6 shows that the results of our benchmark model remain robust: it is optimal for the monopolist who has relatively better product to use limited-time offer to induce some consumer types to visit it first. The intuition is that when the monopolist's product is inferior to all types' outside option in terms of reservation value, it is too costly to compete for prominence. Therefore, the monopolist's optimal strategy is to only sell in the second period and set a high flat price to extract all reservation value: $p = v_A - \frac{s}{\theta_A}$. When the monopolist's product is superior to some types' outside options, the monopolist can use limited-time offers to attract some consumers with bad outside options to visit it first, while leaving the other consumers to explore their outside options first. Doing so makes the monopolist's profit strictly higher.

6 Conclusion

This paper provides a new perspective on analyzing limited-time offer by linking it to consumer search theory. Our idea is inspired by what is happening in the real world: it has become effortless for consumers to obtain price information, and therefore firms could actively advertise prices and discounts to direct a consumer's order of inspecting options. Limited-time offers can thus be an effective tool to gain search prominence. We show how firms can optimally design and use limited-time offers when facing non-strategic and strategic competitors, and what the market equilibrium looks like. In a market with a single firm, it is most profitable for the firm to offer target consumers a limited-time discount as long as the reservation value of its product is higher than that of consumers' outside option. The limited-time discount would induce consumers to sample the product before sampling their outside options. When facing strategic competitors, a general finding is that all firms use limited-time offers, while firms whose products have higher reservation values keep discounts for a shorter period of time. As a result of this tactic, consumers sample the better products earlier than other products, which coincides with the socially optimal search order.

We compare the equilibrium search order under alternative pricing strategies, for example, when limited-time offers are prohibited and only uniform pricing is applied. Equilibrium prices in those cases may lead consumers to sample products with lower reservation values first, which implies a strict decrease in total welfare compared to when limited-time offers are freely used. This finding differs from the findings of the previous literature (Armstrong and Zhou, 2015) due to a new channel highlighted in the paper. That is, limited-time offers can be pro-competitive by endogenously directing the search process to the most efficient order. This finding challenges and complements policy recommendations given by previous research that buy-now discounts should be restricted. We argue that if firms can actively advertise limited-time offers and make prices transparent, policymakers should not prohibit or restrict it.

For future research, it would be interesting to expand the scope of "limited-time offer" beyond price discounts. For example, some advertisements explicitly state that the quantity is limited. Intuitively, such quantity limit works similarly to a limited-time discount. By creating search friction at a later period, i.e., the risk of stock-out, it induces consumers to inspect the deal first before other options. In addition, in some other cases, consumers can observe the status of the product inventory. The lightning deals at Amazon usually include a predetermined inventory quantity, and consumers can see a status bar indicating the percentage of deals that have been ordered. If the status bar progresses fast, consumers might become more optimistic about the product's quality. The quality signaling mechanism goes hand in hand with the search urgency and better attracts consumers to sample the product in advance.

In addition, our theory yields some empirically testable predictions. For example, when competing firms advertise limited-time offers to consumers at the same time (e.g., on a traditional shopping holiday like Black Friday), firms that consumers perceive ex ante as having superior products will hold shorter sales. We hope that our work will inspire further empirical studies to test these predictions to deepen our understanding of limited-time offers.

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Appendix A

Proof of Lemma 1

We extend the proof to a more general setting (footnote 9), where option B is priced by a strategic firm. Lemma 1 is a special case when $p_1^B = p_2^B = 0$.

Without loss of generality, suppose that for a given set of prices $\{(p_1^A, p_2^A), (p_1^B, p_2^B)\}$, the consumer first samples product A, and no matter whether A is a good match or not, she still samples product B; if B is a bad match, the consumer revisits and buys A at p_2^A; if B is a good match, the consumer purchases B at p_2^B. For such search path to exist, we must have $v_B - p_B^2 > v_A - p_A^2$, otherwise the consumer will not sample B when product A is a good match. The consumer's utility from this search path is:

$$-2s + \theta_B(v_B - p_B^2) + (1 - \theta_B)\theta_A(v_A - p_A^2)$$

However, there exists a weakly dominant strategy for the consumer: to sample product B at $t = 1$, if it is a good match, the consumer buys B at $t = 2$; if product B turns out to be a bad match, the consumer continues to sample product A. This strategy yields an expected utility of

$$-s + \theta_B(v_B - p_B^2) + (1 - \theta_B)[\theta_A(v_A - p_A^2) - s]$$

Therefore, if the consumer is satisfied with the first product she has sampled, she will purchase it without further search, and thus recall will never happen.

Proof of Lemma 3

If the pure-strategy equilibrium exists, the product that the consumer samples in the second period must be sold at a price that extracts all the consumer's surplus, otherwise there exists a profitable deviation. Suppose that in equilibrium the consumer samples product A at

$t = 1$, and thus $p^B = v_B - \frac{s}{\theta_B}$. A potential deviation of firm B is to reduce price to attract consumer's search at $t = 1$. To do so, firm B could price lower but not below $(1-\theta_A)(v_B - \frac{s}{\theta_B})$, otherwise firm B makes a strictly lower profit compared to before deviation.

When product B is priced at $(1-\theta_A)(v_B - \frac{s}{\theta_B})$, we solve for product A's price which makes the consumer indifferent between sampling A and B at $t = 1$. Denoted as $\tilde{p}^A$, the price must satisfy the following equation:

$$\theta_A(v_A - \tilde{p}^A) - s + (1-\theta_A)\{\theta_B[v_B - (1-\theta_A)(v_B - \frac{s}{\theta_B})] - s\} =$$
$$\theta_B[v_B - (1-\theta_A)(v_B - \frac{s}{\theta_B})] - s + (1-\theta_B)[\theta_A(v_A - \tilde{p}^A) - s]$$

We get $\tilde{p}^A = v_A - \frac{s}{\theta_A} - \theta_A(v_B - \frac{s}{\theta_B})$. To ensure that firm B charges $p^B = v_B - \frac{s}{\theta_B}$ and would not undercut firm A to change the consumer's search order, the price for product A cannot be higher than $\tilde{p}^A$. Otherwise, firm B has a profitable deviation to a price higher than $(1-\theta_A)(v_B - \frac{s}{\theta_B})$ and makes a higher expected profit.

However, $(\tilde{p}^A, p^B)$ cannot constitute an equilibrium. Firm A will deviate to a higher price approaching $v_A - \frac{s}{\theta_A}$ given that firm B charges $p^B = v_B - \frac{s}{\theta_B}$. As such, a pure-strategy equilibrium with a search path starting from A does not exist. The same logic applies to the proof on the nonexistence of a pure-strategy equilibrium with a search path starting from B. To summarize, no Nash equilibrium exists in pure uniform-pricing strategies.

Mixed-strategy Equilibrium under Uniform Pricing

Having proved that there is no equilibrium in pure strategies, we proceed to show the existence of an equilibrium in mixed strategies. Each firm's strategy is a probability distribution: F_A and F_B with support $[\underline{p}^A, \overline{p}^A]$ and $[\underline{p}^B, \overline{p}^B]$ respectively.

Suppose that the upper boundaries of both supports are the monopoly prices $(v_A - \frac{s}{\theta_A}, v_B - \frac{s}{\theta_B})$, and neither has a mass point at the upper boundary. Firm B can guarantee an expected profit of $\theta_B(1-\theta_A)(v_B - \frac{s}{\theta_B})$ by setting price to $v_B - \frac{s}{\theta_B}$ and selling at $t = 2$.

Therefore, firm B would never price below $(1-\theta_A)(v_B - \frac{s}{\theta_B})$, which can attract the consumer at $t=1$ and yield the same level of expected profit as what is guaranteed. The same logic applies for firm A. Firm A would not price below $(1-\theta_B)(v_A - \frac{s}{\theta_A})$ which yields an expected profit equal to pricing at $v_A - \frac{s}{\theta_A}$.

If $p^A = (1-\theta_B)(v_A - \frac{s}{\theta_A})$ and $p^B = (1-\theta_A)(v_B - \frac{s}{\theta_B})$, the consumer samples A first if $\theta_A(v_B - \frac{s}{\theta_B}) < \theta_B(v_A - \frac{s}{\theta_A})$, which comes from the following equation:

$$\theta_A(v_A - p^A) - s + (1-\theta_A)[\theta_B(v_B - p^B) - s] >$$
$$\theta_B(v_B - p^B) - s + (1-\theta_B)[\theta_A(v_A - p^A) - s]$$

In this scenario, firm A could raise price to $\tilde{p}^A = v_A - \frac{s}{\theta_A} - \theta_A(v_B - \frac{s}{\theta_B})$ which is higher than $p^A = (1-\theta_B)(v_A - \frac{s}{\theta_A})$, and also gain a higher expected profit: $\theta_A\tilde{p}^A$. Therefore, $\tilde{p}^A$ is the lower bound of product A's price distribution, and $p^B = (1-\theta_A)(v_B - \frac{s}{\theta_B})$ is the lower bound of product B's price distribution.

Conversely, if $\theta_A(v_B - \frac{s}{\theta_B}) > \theta_B(v_A - \frac{s}{\theta_A})$, the consumer strictly prefers sampling product B first when $p^A = (1-\theta_B)(v_A - \frac{s}{\theta_A})$ and $p^B = (1-\theta_A)(v_B - \frac{s}{\theta_B})$. Therefore, firm B could raise price to $\tilde{p}^B = v_B - \frac{s}{\theta_B} - \theta_B(v_A - \frac{s}{\theta_A})$ and earn a higher expected profit: $\theta_B\tilde{p}^B$. The following proposition summarizes the properties of mixed-strategy equilibrium:

Proposition 7 *When both firms have to set a uniform price over time, in the mixed-strategy equilibrium:*

1. *when $\theta_A(v_B - \frac{s}{\theta_B}) < \theta_B(v_A - \frac{s}{\theta_A})$, the price distribution of A has support over the interval $[v_A - \frac{s}{\theta_A} - \theta_A(v_B - \frac{s}{\theta_B}), v_A - \frac{s}{\theta_A})$, and the equilibrium profit that firm A obtains must equal its best alternative, namely, $\theta_A(v_A - \frac{s}{\theta_A} - \theta_A(v_B - \frac{s}{\theta_B}))$. The price distribution of B has support over the interval $[(1-\theta_A)(v_B - \frac{s}{\theta_B}), v_B - \frac{s}{\theta_B}]$, and the equilibrium profit that firm B obtains is $\theta_B(1-\theta_A)(v_B - \frac{s}{\theta_B})$.*

2. *when $\theta_A(v_B - \frac{s}{\theta_B}) > \theta_B(v_A - \frac{s}{\theta_A})$, the price distribution of A has support over the inter-*

val $[(1-\theta_B)(v_A - \frac{s}{\theta_A}), v_A - \frac{s}{\theta_A}]$*, and the equilibrium profit that firm A obtains is* $\theta_A(1-\theta_B)(v_A - \frac{s}{\theta_A})$*. The price distribution of B has support over the interval* $[v_B - \frac{s}{\theta_B} - \theta_B(v_A - \frac{s}{\theta_A}), v_B - \frac{s}{\theta_B})$*, and the equilibrium profit that firm B obtains is* $\theta_B(v_B - \frac{s}{\theta_B} - \theta_B(v_A - \frac{s}{\theta_A}))$*.*

Proof of Proposition 5

The proposition consists of two parts, so our proof follows this structure. First, we will show that all equilibria are socially efficient. Second, we prove that, when firm 1 to N can be FOSD ranked, the proposed equilibrium is indeed a profit-dominant equilibrium.

Part 1: All equilibria are socially efficient

What we need to show is that in any equilibrium, for any $i < j$, product i is sampled earlier than product j. We prove this by contradiction. Suppose there exists an equilibrium with prices $\{p^i_t\}_{i,t=1\ldots N}$ and the consumer's search order is socially sub-optimal. This implies that there must exist two firms m and n, such that $RV_m > RV_n$, and the consumer samples product j at time $\hat{t}$ and product i at time $\hat{t}+1$.

To sustain an equilibrium, three conditions have to hold simultaneously. First, it must be optimal for the consumer to sample product m at time $\hat{t}+1$, instead of completely ignoring product m and sampling other products that have not been sampled yet. Let $\mathcal{I}_t$ denote the set of firms in equilibrium that are sampled at periods $t, t+1\ldots N$. Let $\bar{W}(\{p^i_t\}_{i\in\mathcal{I},t\geq\bar{t}})$ denote the *highest* payoff that the consumer gains if she only faces prices of firms in the set $\mathcal{I}$ during periods $\bar{t}, \bar{t}+1, \ldots, N$. Then, this condition implies:

$$\theta_m(v_m - p^m_{\hat{t}+1}) - s + (1-\theta_m)\bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2},t\geq\hat{t}+2}) \geq \bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2},t\geq\hat{t}+1}) \tag{6}$$

Second, firm m must find it unprofitable to deviate to a price plan of $(\tilde{p}^m_{\hat{t}}, \tilde{p}^m_{t\neq\hat{t}} = RV_m)$ to compete for selling at time $\hat{t}$, such that the consumer is just indifferent between sampling product m at time $\hat{t}$ and product n at time $\hat{t}$. The highest possible $\tilde{p}^m_{\hat{t}}$ is determined by:

$$\theta_m(v_m - \tilde{p}^m_{\hat{t}}) + (1-\theta_m)\bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2}, t\geq\hat{t}+1}) = \theta_n(v_n - p^n_{\hat{t}}) + (1-\theta_n)\bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2}, t\geq\hat{t}+1})$$

This implies:

$$\theta_m\tilde{p}^m_{\hat{t}} = \theta_m v_m - \theta_n(v_n - p^n_{\hat{t}}) - (\theta_m - \theta_n)\bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2}, t\geq\hat{t}+1})$$

To ensure that this deviation is unprofitable, firm m must make a higher profit by selling at $p^m_{\hat{t}+1}$ at time $\hat{t}+1$. We need $\tilde{p}^m_{\hat{t}} \leq (1-\theta_n)p^m_{\hat{t}+1}$. That is:

$$(1-\theta_n)\theta_m p^m_{\hat{t}+1} \geq \theta_m v_m - \theta_n(v_n - p^n_{\hat{t}}) - (\theta_m - \theta_n)\bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2}, t\geq\hat{t}+1}) \tag{7}$$

Third, firm n must find it unprofitable to deviate to a price plan of $(\tilde{p}^n_{\hat{t}+1}, \tilde{p}^n_{t\neq\hat{t}+1} = RV_n)$ to compete for selling at time $\hat{t}+1$, such that the consumer is just indifferent between sampling product m at $\hat{t}$ and n at $\hat{t}+1$, and, sampling product m at $\hat{t}$ and firms in the set $\mathcal{I}_{\hat{t}+2}$ since time $\hat{t}+1$. The highest possible $\tilde{p}^n_{\hat{t}+1}$ is determined by:

$$\theta_n(v_n - \tilde{p}^n_{\hat{t}+1}) + (1-\theta_n)\bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2}, t\geq\hat{t}+2}) = \bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2}, t\geq\hat{t}+1})$$

This implies:

$$\theta_n\tilde{p}^n_{\hat{t}+1} = \theta_n v_n + (1-\theta_n)\bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2}, t\geq\hat{t}+2}) - \bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2}, t\geq\hat{t}+1})$$

To ensure that this deviation is unprofitable, firm n must make a higher profit by selling at $p^n_{\hat{t}}$ at time $\hat{t}$. This implies that $p^n_{\hat{t}} \geq (1-\theta_m)\tilde{p}^n_{\hat{t}+1}$. Thus, we have:

$$\theta_n p^n_{\hat{t}} \geq (1-\theta_m)\theta_n v_n + (1-\theta_m)(1-\theta_n)\bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2}, t\geq\hat{t}+2}) - (1-\theta_m)\bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2}, t\geq\hat{t}+1}) \tag{8}$$

Substitute Inequality (8) into (7), we have:

$$\theta_m(v_m - p^m_{\hat{t}+1}) - s + (1-\theta_m)\bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2},t\geq\hat{t}+2}) - \bar{W}(\{p^i_t\}_{i\in\mathcal{I}_{\hat{t}+2},t\geq\hat{t}+1}) \leq \frac{\theta_m\theta_n}{1-\theta_n}(RV_n - RV_m) - \frac{1-\theta_m}{1-\theta_n}s \quad (9)$$

Inequality (7) implies that the LHS of (9) is positive. Since $RV_m > RV_n$, the RHS of (9) is strictly negative. Therefore, we find a contradiction.

Part 2: Characterizing the profit-dominant equilibrium

We divide this proof into three steps. First, given the proposed prices, the consumer's best response is to search in the order from firm 1 to firm N . Second, for any firm i, there exists no profitable deviation. The first two steps prove that the strategies summarized in Proposition 5 do constitute an equilibrium. The final step proves that this equilibrium is profit-dominant. We start with the following lemma.

Lemma 4 *Regardless of prices, searching from firm 1 to firm N is socially optimal. When transaction prices are* $p^i_i = RV_i - \sum_{k=i+1}^N \left[\theta_k RV_k \Pi_{m=1}^{k-1}(1-\theta_m)\right], \forall i = 1, 2, ..., N$, *each firm earns all the additional surplus it brings to the economy.*

Proof. The first statement is a natural extension from the $N = 2$ case in the previous section. The total surplus of having N firms in the market, denoted by TS_N, equals:

$$TS_N = \sum_{j=1}^{N}\left[(\theta_j v_j - s)\Pi_{k=1}^{j-1}(1-\theta_k)\right] \quad (10)$$

If firm i is missing, the total surplus of having the remaining $N-1$ firms, denoted by TS_{N-i}, equals:

$$TS_{N-i} = \sum_{j=1}^{i-1}\left[(\theta_j v_j - s)\Pi_{k=1}^{j-1}(1-\theta_k)\right] + \sum_{j=i+1}^{N}\left[(\theta_j v_j - s)\Pi_{k=1}^{i-1}(1-\theta_k)\Pi_{m=1}^{j-1}(1-\theta_m)\right]$$

And therefore, the surplus created by firm i's entry, Δ_i, is:

$$\Delta_i = TS_N - TS_{N-i} = \theta_i RV_i - \theta_i \sum_{k=i+1}^{N} \left[\theta_k RV_k \Pi_{m=1}^{k-1}(1-\theta_m)\right] \tag{11}$$

It is easy to verify that $\Delta_i = \theta_i p_i^i$. Hence, we show that each firm extracts all the additional surplus it brings to the economy. ■

Step 1: Consumer's optimal search order

The proposed pricing strategies are: for any firm $i < N$, $p_t^i = RV_i, \forall t > i$; $p_i^i = RV_i - \sum_{k=i+1}^{N}\left[\theta_k RV_k \Pi_{m=1}^{k-1}(1-\theta_m)\right]$; $p_t^i = (1-\theta_{i-1})p_i^i, \forall t < i$; for firm N, $p_N^N = RV_N$; $p_t^N = (1-\theta_{N-1})RV_N, \forall t < N$. In the first step, we need to show that under such price plans, it is optimal for the consumer to sample from firm 1 to firm N.

Under such price plans, if the consumer samples from firm 1 to firm N, her surplus is:

$$W_{1\to\ldots\to N} = TS_N - \sum_{i=1}^{N}\Delta_i$$

where TS, defined by Equation (10), is the total surplus when the search order is from firm 1 to N; and Δ_i, defined by Equation (11), is both the additional surplus firm i contributes to the economy and the profit that firm i makes.

Now, consider any other order in which consumers search through firms. Suppose M of the N firms have prices that are not RVs at the period of being sampled. Let $\mathcal{M}$ denote the set of these M firms. The consumer surplus under the new search order equals the total surplus with the M sampled firms, denoted by $\tilde{TS}_M$, minus the profits that these M firms make, denoted by $\tilde{\Pi}_i$ for each firm $i \in \mathcal{M}$.

First, it is easy to verify that $\tilde{TS}_M \leq TS_M$, i.e., the total surplus with the M firms is upper-bounded by the total surplus when the consumer samples products from 1 to N. This is directly obtained from Lemma 4. And second, $\tilde{\Pi}_i \geq \Delta_i, \forall i \in \mathcal{M}$. That is, for the firms sampled at non-RV prices, their profits under the new search order must be (weakly) higher than their profits under the search order from 1 to N. Under the prices summarized in Proposition 5, if firm i is sampled at time i, its demand is $(1-\theta_1)\cdot(1-\theta_2)\cdots(1-\theta_{i-1})$ (the probability that all the previous $i-1$ firms' products are not good fits); and if the firm's product is a good match for the consumer (the probability is θ_i), the product is sold at price p_i^i. If the firm is sampled at time $i' < i$, its profit is $\frac{1-\theta_{i-1}}{(1-\theta_{i'})\cdot(1-\theta_{i'+1})\cdots(1-\theta_{i-1})} \geq 1$ times higher.

Combining the above two points, we know that $\tilde{TS}_M \leq TS_M$ and $\tilde{\Pi}_i \geq \Delta_i, \forall i \in \mathcal{M}$. Then the consumer surplus, $\tilde{W}_M = \tilde{TS}_M - \sum \tilde{\Pi}_i$ has to be lower than the consumer surplus when sampling products from 1 to N.

Step 2: There exists no profitable deviation

In the second step, we will show that for any firm i, there exists no profitable deviation to any other price plan.

Given any deviation of firm i, the worst-case scenario for the consumer is *as if* firm i is removed from the economy. First, assume firm i is removed from the economy and the strategies of all other firms remain unchanged. From step 1, we know that the optimal search order is now to sample firm 1 to firm $i-1$ at period 1 to period $i-1$ respectively, and then to sample from firm $i+1$ to firm N sequentially starting from period i. From step 1, we also know that the profit of each firm $i' > i$ stays the same, as its demand is multiplied by $\frac{1}{(1-\theta_i)}$, while the transaction price is $(1-\theta_i)$ of the previous one. And for each firm $i' < i$, its demand and transaction price remain unchanged.

Second,from Lemma 4, we know that $\Pi_i = \Delta_i$. That is, firm i's profit is equal to the additional value it contributes to the economy. Therefore, $TS_{N-i} = TS_N - \Pi_i$. Since all the other $N-1$ firms make the same profits in the absence of firm i, the consumer will have the same surplus when firm i is removed from the economy. Next, we will discuss three possible deviations for any firm i.

Deviation 1: firm i may deviate to a higher price $\tilde{p}_i^i > p_i^i$ but still sells at period i. This cannot be a profitable deviation, as the consumer would rather skip sampling firm i and instead sample product $i+1$ at period i, product $i+2$ at period $i+1$, and so on, which guarantees the same consumer surplus as when there is no deviation.

Deviation 2: firm i may deviate to induce itself to be sampled at period $t > i$ and sell its product at price $\tilde{p}^i_t$. Then, it is optimal for the consumer to sample product $i+1, i+2, \ldots, t$ at period $i, i+1, \ldots, t-1$, and product i at time t. As for any firm $i' \neq i$, it makes the same profit as before the deviation, but the total social surplus under the new search order is smaller than the $1 \to 2 \to \cdots \to N$ order, firm i's profit must be lower after the deviation, otherwise, the consumer would simply skip firm i.

Deviation 3: firm i may deviate to induce itself to be sampled at period $t < i$ and sell its product at price $\tilde{p}^i_t$. Note that at the original price p^i_t and being sampled by consumer at period t, firm i makes the same profit as when it is sampled at period i and sells the product at price p^i_i. Thus, a profitable deviation must be such that firm i sells at $\tilde{p}^i_t > p^i_t$ at period t. However, such deviation can never induce firm i to be sampled at period t. For such deviation to hold, the consumer surplus of sampling product i at time t must exceed the surplus of skipping product i, which equals the consumer surplus under the original prices and $1 \to 2 \to \cdots \to N$ search order. However, since $\tilde{p}^i_t > p^i_t$, the $1 \to 2 \to \cdots \to N$ order can never be optimal for the consumer under the original prices. Hence, we obtain a contradiction.

Step 3: Profit-Dominant Equilibrium

Since we have shown that each firm i extracts all the surplus it brings to the economy, each firm i achieves the highest possible surplus in all equilibria.

Proof of Proposition 6

This proposition consists of two statements. Part 1 is if $\forall (v,\theta) \in supp(f)$, $v_A - \frac{s}{\theta_A} \leq v - \frac{s}{\theta}$, limited-time offers cannot make strictly higher payoff than uniform pricing. Part 2 is if $\forall (v,\theta) \in \mathcal{B}$, $v_A - \frac{s}{\theta_A} > v - \frac{s}{\theta}$, limited-time offers are strictly better than uniform pricing. We will prove the two parts separately.

To begin with, note that conditional on the consumer investigating the outside option first, it is always optimal to set a price of $v_A - \frac{s}{\theta_A}$ at $t = 2$. Thus, under uniform pricing, the monopolist optimally sets price $\hat{p} = v_A - \frac{s}{\theta_A}$ to induce all types of consumers to sample A at $t = 2$, and makes an expected profit of $\hat{\pi} = (v_A - \frac{s}{\theta_A}) \int\int (1-\theta) f(v,\theta) dv d\theta$. For limited-time offers, it is obvious that all price paths are weakly dominated by setting $p_2 = v_A - \frac{s}{\theta_A}$. On one hand, this gives the highest incentive for the consumer to sample A first; because if the consumer chooses $B \to A$ search path, she gets zero expected payoff if B is a bad match. On the other hand, for the consumer with relatively good outside option, $p_2 = v_A - \frac{s}{\theta_A}$ extracts the highest surplus from consumers following $B \to A$ path. Therefore, in the optimal limited-time offer $(\bar{p}_1, \bar{p}_2)$, we must have $\bar{p}_2 = v_A - \frac{s}{\theta_A}$.

Part 1: we prove by contradiction. Assume there exists a limited-time offer $(p_1, p_2 = v_A - \frac{s}{\theta_A})$ that gives strictly higher payoff than that under optimal uniform pricing, $\hat{\pi}$. Note that p_1 must be low enough such that some consumer types will sample A first, otherwise profit would be exactly the same. For a consumer with type (v_B, θ_B), she samples product A first if and only if:

$$\theta_B v_B - s \leq \theta_A(v_A - p_1) - s + (1-\theta_A)(\theta_B v_B - s) \Longleftrightarrow \theta_B v_B \leq v_A - \frac{s}{\theta_A} + s - p_1 \tag{12}$$

Selling to a consumer of type (v_B, θ_B) at price p_1 at $t = 1$ generates a strictly higher profit than selling at $p_2 = v_A - \frac{s}{\theta_A}$ at $t = 2$ if and only if:

$$p_1 > (1-\theta_B)(v_A - \frac{s}{\theta_A}) \Longleftrightarrow v_A\theta_B - \frac{\theta_B s}{\theta_A} + s > v_A - \frac{s}{\theta_A} + s - p_1 \tag{13}$$

Combine (12) and (13), we have:

$$v_A\theta_B - \frac{\theta_B s}{\theta_A} + s > \theta_B v_B \Longleftrightarrow v_A - \frac{s}{\theta_A} > v_B - \frac{s}{\theta_B} \tag{14}$$

and (14) contradicts with the assumption that $\forall(v,\theta) \in supp(f), v_A - \frac{s}{\theta_A} \leq v - \frac{s}{\theta}$.

Part 2: we construct a limited-time offer (unnecessarily optimal) that gives a strictly higher profit to the monopolist than the optimal uniform price profit $\hat{\pi}$. Let $\underline{U}$ be the smallest θv among low types consumers. Then a limited-time offer $(p_1 = v_A - \frac{s}{\theta_A} - (\underline{U} - s), p_2 = v_A - \frac{s}{\theta_A})$ is strictly more profitable than a uniform price $v_A - \frac{s}{\theta_A}$.

For a consumer of type (v_B, θ_B), she samples A first if and only if $\theta_B v_B \leq v_A - \frac{s}{\theta_A} + s - p_1$. Thus, under $(p_1 = v_A - \frac{s}{\theta_A} - (\underline{U} - s), p_2 = v_A - \frac{s}{\theta_A})$, all consumers with type $\theta v = \underline{U}$ will sample product A in the first period. It is strictly more profitable for the monopolist to sell to these consumers, as at $p_1 = v_A - \frac{s}{\theta_A} - (\underline{U} - s)$, it is easy to verify that Equation (13) always holds.

Note that this price path is unnecessarily the optimal for the monopolist. This is the highest price that makes the consumer who has the worst outside option to visit the monopolist first. By further reducing p_1, the monopolist can attract more consumer types to sample product A at $t = 1$, but at the cost of losing profits from lower types and making some unfavorable consumers (those with types (v_B, θ_B) such that $v_A - \frac{s}{\theta_A} \geq v_B - \frac{s}{\theta_B}$) visit the firm first.